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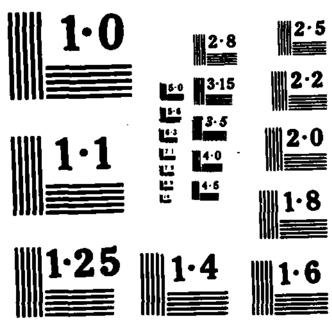
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SACLANT ASW RESEARCH CENTRE REPORT

PASSIVE SONAR DATA PROCESSING, PROPAGATION MODELS AND THE DATA CROSSCORRELATION MATRIX: A SURVEY

by

Even B. LUNDE

Walter M.X. ZIMMER

1 MARCH 1985

NORTH ATLANTIC TREATY ORGANIZATION

SACLANTCEN LA SPEZIA, ITALY

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This report has been prepared as part of Project 02.

APPROVED FOR DISTRIBUTION

Ralph R. Goodman

RALPH R. GOODMAN

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PASSIVE SONAR DATA PROCESSING, PROPAGATION MODELS AND THE DATA CROSSCORRELATION MATRIX - A SURVEY

by

Even B. Lunde and Walter M. Zimmer

ABSTRACT

The report follows one possible development in the processing of data from passive-sonar arrays: from robust processing of stationary data using simple models, to refined processing of data acquired under non-stationary conditions, using more complex models. For such a development to take place, it is pointed out that a strong interaction between researchers in propagation modelling and those concerned with array processing seems necessary. As the level of ambition related to information extraction from the sound field is increased, some of the most common assumptions made in the processing of the data, such as stationarity, plane wavefront propagation model, etc., have to be questioned. If these assumptions do not survive, the question will be how to reformulate them and how to structure new models in such a way that they can be incorporated in the data processing (information extraction) so as to improve sonar performance.

INTRODUCTION

Let us start out with some definitions:

X : Data column vector with N elements, where the elements are some representation of the outputs from the N hydrophones of a towed array, being part of a passsive sonar system.

R : $E\{X X^*\}$: The expected cross-correlation matrix of the data vector X. Without loss of generality, it is assumed that $E\{X\}=0$. $E\{\ \}$ is the expectation operator, i.e. ensemble averaging operator. * is the complex conjugate transpose operator.

In a surprisingly large part of all passive sonar systems, processing on the data vector X involves, explicitly or implicitly, the cross-correlation matrix R, or more correctly, its estimate, $\hat{R}$.

As an example, a weighted summation of the elements of X is often performed (beamforming)

$$y = \Sigma w_i^* x_i = W^* X \quad ,$$

followed by an averaging of the squared absolute value, the power, of y:

$$P = E\{|y|^2\} E\{yy^*\} = E\{W^*XX^*W\}$$

$$= W^*E\{XX^*\}W = W^*RW$$

Indeed, nothing but the cross-correlation matrix R is involved.

The two main reasons for the extensive concentration on the R matrix are:

- The probability density of the data vector X is often assumed to be gaussian (normal) with zero mean value. Hence the density is completely described by the cross-correlation matrix R .

- Many models, criteria, and principles used in the data processing are such that only first and second order statistics are asked for. Typical for this group is linear least-mean-squares processing.

Using the cross-correlation matrix in array processing often involves two distinct steps.

- Estimation of the cross-correlation matrix itself.

- Using the matrix to extract information from the data X or derived quantities, like R itself.

These two steps are usually made independently, even if this might not be optimal [1].

The basic assumption in estimating R is stationarity (or quasi-stationarity), such that instead of expectation or ensemble averaging, time averaging can be used as a substitute. This rather crude assumption leads surprisingly often to satisfactory results. The reasons must be that in some (few ?) cases, the process in question is indeed stationary, or the employed data processing is robust enough to tolerate faulty assumptions. In the present report, we will start out from the safe position of robust processes on stationary data and end with more refined modelling where lack of stationarity will highly influence the end product or the processing strategy (as it happens, this is also the chronological order). It seems, also, that we might be heading for a future abandonment of the matrix, which we may or may not consider to be disgraceful. To follow this development is the first purpose of this report.

This report does not deal explicitly with the details of estimating the matrix, but assumes that the desired estimate is available. This might be of the matrix itself, R, some function of it, such as the inverse R^{-1} [2], or an estimate constrained to have a certain structure, typically a Toeplitz structure [3]. The explicit estimate itself, $\hat{R}$, often has a heavy influence on the quality of the ensuing processing.

Instead, we concentrate on the task of extracting information from the sonar data through array (spatial) processing. In order to do that, we need a sound-propagation model. And that leads directly to the second purpose of this report: to throw some light on the relation between propagation-model research(ers) and array-processing research(ers). According to [4] the two disciplines can be defined respectively as:

- To improve the representation of the sound field in order to predict its value at a point P for a source at point S .

- To refine data processing in order to discriminate different sources and to determine their individual signals together with their individual position.

It goes without saying that a strong degree of interaction between researchers of the two disciplines is highly desirable. This should include groups acquiring enough interdisciplinary knowledge (and interest) to trigger discussion fruitful to both disciplines. To improve sonar performance by improving data processing could (and should ?) be the main task of propagation modelling. Today it is mainly delegated to the (humble) task of transmission-loss predictions, and in the same vein, evaluation from an operational point of view of a new sonar-system concept. But improved propagation modelling is not often part of the new concept. One wonders why !

In this report we start, as an example, with the common plane-wavefront propagation model, and later refine it to the so-called normal-mode model [5]. We then study the consequences that this might have, both on the processing itself, and on the amount of information extracted from the data.

The report makes no claim in any sense to be complete or authoritative. Its main purpose is to encourage two (independent) lines of thought provoked by the questions:

- How healthy are some of the assumptions made in the data processing? Will (and should) they all survive as more refined models are applied ?

- How can (and should) more refined sound-propagation models be formulated, such that they can be applied to the processing of sonar data so as to extract more information ?

1 ASSUMPTIONS

1.1 Frequency bandwidth

The data are assumed to be narrowband-filtered with bandwidth B, shifted down to zero frequency (quadrature bandpass filtering). The two main reasons for this are:

- A delay τ can be implemented by multiplication with a phase rotating factor, $\exp\{-j2\pi f\tau\}$, as long as the delay/bandwidth product is much smaller than unity, $\tau \cdot B \ll 1$, where f is the centre frequency of the band.

- If a stationary, random process is observed for a time that is large compared with its correlation time, its complex fourier coefficients are uncorrelated.

These two properties often simplify implementation – such as implementing the delay necessary in beamforming, where the maximum delay is the travel time of sound across the array – and allow separate treatment of the different frequency bands.

1.2 Receiver system

Our receiver system is a towed, horizontal array with equidistant hydrophones (Fig. 1). The distance between neighbouring hydrophones is Δx and the total number is N.

Both spherical and Euclidean coordinate systems are introduced (Fig. 1). The first hydrophone (number 0) is the reference point, except for the depth coordinate, which is zero at the surface:

$$x = R\cos\beta \qquad y = R\sin\beta\cos\psi \qquad z = Z_r + R\sin\beta\sin\psi$$

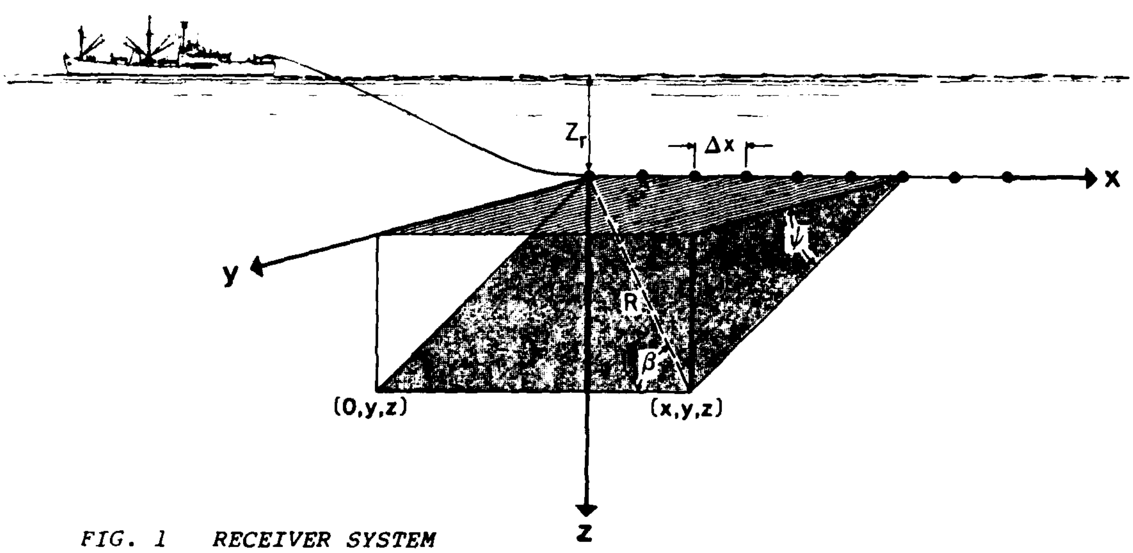

FIG. 1 RECEIVER SYSTEM

2 PROPAGATION MODELS

We deal with two models only: the standard plane-wavefront propagation model, and the normal-mode model.

2.1 Plane-wavefront model

This is both the simplest and most common model. It is the tangential approximation to the spherical spreading of sound from a point source. If this tangential plane is parallel to the y-z plane, we can express the sound pressure by:

$$p(r,t_n) = s_n \cdot e^{-jkr}$$
$$k = 2\pi\ \lambda = 2\pi f/c \quad ,$$

where

s	=	stochastic process slowly changing
k	=	wavenumber
λ	=	wavelength
f	=	centre frequency
c	=	sound speed
r	=	distance (displacement) orthogonal to the plane wavefront.

The plane-wavefront approximation is assumed valid over a distance d in the plane satisfying, $\lambda \cdot r > d^2$, the so-called far-field condition, where r is the distance from the source.

Combining this model with the receiver array of Sect. 1.2 yields:

$$X(t_n) = \begin{Bmatrix} \cdot \\ p_i \\ \cdot \end{Bmatrix} = s_n \begin{Bmatrix} \cdot \\ e^{-jkx_i \cos\beta} \\ \cdot \end{Bmatrix} = s_n \cdot D(k \cos\beta) \quad ,$$

where

D	=	source vector
x_i	=	position of hydrophone i
β	=	direction of incoming plane wavefront.

2.2 Normal-mode model

The normal-mode model [5] is a natural refinement to the traditional plane-wave model. For a small array system, the modes are indistinguishable, while they will be more and more noticeable with increasing array size. Using the simplest possible notation, we express the sound pressure as:

$$p(r,t_n) = s_n \sum_{m=1}^{M} b_m\, e^{-jk_m r} \quad ,$$

where

k_m	=	horizontal wavenumber
r	=	horizontal displacement
b_m	=	modal weight, being a function of source depth, receiver depth, and range.

Combining this with our receiver array yields:

$$X(t_n) = s_n \sum_{m=1}^{M} b_m \left\{ e^{-jk_m x_i \cos\beta} \right\} = s_n \sum_{m=1}^{M} b_m \; D(k_m \cos\beta) \quad ,$$

$$= s_n \sum_{m=1}^{M} b_m \; D(k \cos\beta_m) \quad = s_n \sum_{m=1}^{M} b_m \; D(\beta_m) \quad ,$$

where

$$\cos\beta_m = \frac{k_m}{k} \cos\beta \quad .$$

This expression says that the normal-mode model can be interpreted as a weighted combination of plane waves, each with slightly different direction, dependent on the mode's horizontal wavenumber, k_m. It is worth noticing that k_m is a function only of the local channel (medium).

3 PROCESSING WITH PLANE-WAVEFRONT MODEL

In this chapter we study the connection between the plane-wavefront model and the cross-correlation matrix. It is natural to introduce the concept of sound-field density. Given the matrix, what information is available about the sound-field density, and can it be estimated uniquely? Conventional, adaptive, and maximum-entropy beamforming give different answers. We will see why.

3.1 Cross-correlation matrix and the sound field

According to Sect. 2.1, the array datum from one point source is:

$$X = s_n \cdot D(k \cos\beta) \quad . \tag{Eq. 1}$$

The contribution to the cross-correlation matrix would be:

$$\begin{aligned} R &= E\{XX^*\} = E\{s_n \cdot s_n^* \, D(\beta)\cdot D^*(\beta)\} \\ &= E\{s_n \cdot s_n^*\} \, D(\beta)\cdot D^*(\beta) = \sigma^2(\beta)\cdot D(\beta)\cdot D^*(\beta) \quad , \end{aligned} \tag{Eq. 2}$$

where $E\{\ \}$ is the expectation operator and s_n is assumed to have zero mean. Equation 2 can be generalized to include independent contributions from all directions:

$$\sigma^2(\beta) \rightarrow P(\beta,\psi) \sin\beta \, d\beta \, d\psi \tag{Eq. 3}$$

$$R = \int_\beta d\beta \int_\psi d\psi \, P(\beta,\psi) \sin\beta \, D(\beta) \, D^*(\beta) \quad , \tag{Eq. 4}$$

where $P(\beta,\psi)$ is the sound-field density. Obviously, as $D(\beta)$ is independent of ψ, the horizontal line array integrates over ψ:

$$R = \int_\beta d\beta \sin\beta \, P(\beta) \, D(\beta) \, D^*(\beta) \quad , \tag{Eq. 5}$$

where

$$P(\beta) = \int_0^\pi P(\beta,\psi) \, d\psi \quad . \tag{Eq. 6}$$

chosen so that the model matrix R fits as closely as possible to the estimated matrix $\hat{R}$ (from data). As R has a Toeplitz structure, $\hat{R}$ can be estimated while being constrained to such a structure [3]. In that case $\hat{R}$ provides 2N-1 real data values:

$$r_k \;, \quad |k| \leq N, \text{ with } r_k = r^*_{-k}.$$

Hence, to determine all the parameters requires that $K + T \leq N$

One approach to this estimation problem is to attempt to estimate the continuous part first. As, by definition, the discrete part should produce a matrix R_d of rank $T \leq N-K$, the parameters a_k, $|k| < K$, should be chosen such that the K smallest eigenvalues of R - Rc are as close to zero as possible. The estimation criterion in [9] is in agreement with this principle:

$$\min_B J = \sum_{n=N-K}^{N-1} \lambda_n^2 \quad , \tag{Eq. 47}$$

$$\hat{R}_d = \hat{R}-R_c(B) = \hat{R} - \sum_{k=1-K}^{K-1} b_k I_k = \sum_{n=0}^{N-1} \lambda_n E_n E^*_n \quad , \tag{Eq. 48}$$

where

$$B = [b_o, b_1, \ldots, b_{K-1}]^T \tag{Eq. 49}$$

λ_n = Eigenvalues of $\hat{R} - R_c(B)$

E_n = Eigenvectors of $\hat{R} - R_c(B)$.

Details about the algorithm are given in App. A.

The next step is to estimate σ_i^2 and v_i, (i=0, ..., T-1) from $\hat{R}_s$, which is now identical to the same problem in the Pisarenko method; again the goniometer method is one candidate [8].

If we apply this method to our old example of one point source and white noise, we of course obtain the correct answer (our example is chosen to fit the model !)

3.2 Sensitivity to Source-Vector Model

All the methods outlined in the previous sections assumed a plane-wave propagation model, leading to the classical steering vector as a model of the source vector.

Comparison with Eq. 36 yields $T = N-1$. At this stage, several methods can be used to estimate σ_i^2 and v_i, $i = 1,N-1$. Among them is the so-called goniometer method [8].

In order to allow a more general continuous term, the authors have attempted to generalize the Pisarenko method [9]. One possibility is to view Eq. 37 as the zero term in a fourier series. A generalized version is then obviously:

$$P_c(v) = \sum_{k=1-K}^{K-1} a_k\, e^{-j2\pi kv} \quad , \qquad \text{(Eq. 42)}$$

such that

$$b_{k,c} = a_k \; , \quad |k| < K$$
$$= 0 \; . \quad |k| \geq K$$

The matrix is:

$$R_c = \sum_{k=1-K}^{K-1} a_k\, I_k \quad ,$$

where I_k is the zero matrix, except that diagonal number k has elements of value one. For $K = 1$ this corresponds to Pisarenko method.

To summarize, the structure is:

$$P(v) = \sum_{i=o}^{T-1} \sigma_i^2\, \delta(v-v_i) + \sum_{k=1-K}^{K-1} a_k\, e^{-j2\pi kv} \quad , \qquad \text{(Eq. 43)}$$

$$b_k = \begin{cases} \sum_{i=o}^{T-1} \sigma_i^2\, e^{j2\pi k v_i} + a_k & |k| < K \qquad \text{(Eq. 44)} \\ \sum_{i=o}^{T-1} \sigma_i^2\, e^{j2\pi k v_i} & |k| \geq K \qquad \text{(Eq. 45)} \end{cases}$$

$$R = \sum_{i=o}^{T-1} \sigma_i^2\, D(v_i)\, D^*(v_i) + \sum_{k=1-K}^{K-1} a_k\, I_k \quad , \qquad \text{(Eq. 46)}$$

where the first term in all these expressions is the discrete one (signal part) and the second term is the continuous one (noise part). Altogether there are $2K-1 + 2T$ unknown real parameters ($a_k = a^*_{-k}$). They have to be

It is quite important in the following that the rank of R_d is given by:

$$\text{rank}\ (R_d) = \min(T,N) \quad , \qquad \text{(Eq. 36)}$$

where N is the dimension of R_d and the number of hydrophones. The orthogonal-decomposition method will work only if this rank is less than N; in other words, if R_d is a singular matrix. This means that only $T < N$ discrete lines can be allowed in the model of $P(v)$.

Turning attention to the continuous term $P_c(v)$, it is not so obvious or straight-forward how to model P_c or R_c. Mermoz [7] indicates that general plausible models of noise turbulence could be used to formulate R_c as a function of several free parameters A (A is a parameter vector):

$$R_c = R_c(A) \quad .$$

and that A could be calibrated (chosen) in such a way that the matrix $R_d = R - R_c$ has minimum rank. If the rank is larger than the number of free parameters in R_c, it indicates a good noise model. Bienvenu and Kopp [8] work along these lines.

Another favourite is the Pisarenko model, which has only one parameter:

$$P_c(v) = \sigma_c^2 \quad . \qquad \text{(Eq. 37)}$$

The motivation is that $P_c(v)$ contains uncorrelated noise in the array data. Note that in this case:

$$R_c = \sigma_c^2\ I \quad , \qquad \text{(Eq. 38)}$$

$$R = R_d + R_c = \sum_{i=o}^{T-1} \sigma_i^2\ D(v_i)\ D^*(v_i) + \sigma_c^2\ I \quad , \qquad \text{(Eq. 39)}$$

or

$$R_d = R - R_c = \sum_{n=1}^{N} (\lambda_n - \sigma_c^2)\ E_n\ E_n^* \quad , \qquad \text{(Eq. 40)}$$

where λ_n and E_n are the eigenvalues and eigenvectors of R.

In order to give R_d a rank less than N, σ_c^2 is chosen to be equal to the smallest eigenvalue of R, λ_{ni}, resulting in:

$$R_d = \sum_{n=1}^{N-1} (\lambda_n - \sigma_c^2)\ E_n\ E_n^* \quad , \qquad \text{(Eq. 41)}$$

$$\text{rank}\ (R_d) = N-1 \quad .$$

It is worth mentioning that the constraints (Eq. 30) can be disguised, or made somewhat different, without changing the principle. The so-called WB2 algorithm uses the following constraints [6]:

$$P(u_k) = D^*(u_k)\ R\ D(u_k) = \int_{-\frac{1}{2}}^{\frac{1}{2}} dv\ P(v)\ \left|D^*(u_k)\cdot D(v)\right|^2 \quad . \qquad \text{(Eq. 31)}$$

This means that the WB2 algorithm has replaced $\exp\{-j2\pi kv\}$ by the "beampattern" $|D^*(u_k)\ D(v)|^2$ as the weighting function in the functional of Eq. 31.

3.1.5 Continuous plus discrete decomposition (Orthogonal decomposition method)

Because of the limited size of the array, all the previous methods have some problems with estimating the discrete point source in the sound-field density. One way to improve that is to allow for a finite number of discrete lines in the estimation of the density, such that $P(v)$ consists of a discrete and a continuous term:

$$P(v) = P_d(v) + P_c(v) \quad . \qquad \text{(Eq. 32)}$$

Keeping the plane-wave model, the discrete term is easy to structure

$$P_d(v) = \sum_{i=0}^{T-1} \sigma_i^2 \cdot \delta(v-v_i) \quad , \qquad \text{(Eq. 33)}$$

where T is the number of sources.

The contribution to the fourier-series coefficient is:

$$b_{k,d} = \int_{-\frac{1}{2}}^{\frac{1}{2}} P_d(v)\ e^{j2\pi kv}\ dv = \sum_{i=0}^{T-1} \sigma_i^2\ e^{j2\pi kv_i} \quad , \qquad \text{(Eq. 34)}$$

and to the cross-correlation matrix is:

$$R_d = \int_{-\frac{1}{2}}^{\frac{1}{2}} P_d(v)\ D(v)\ D^*(v)\ dv = \sum_{i=0}^{T-1} \sigma_i^2\ D(v_i)\cdot D^*(v_i) \quad . \qquad \text{(Eq. 35)}$$

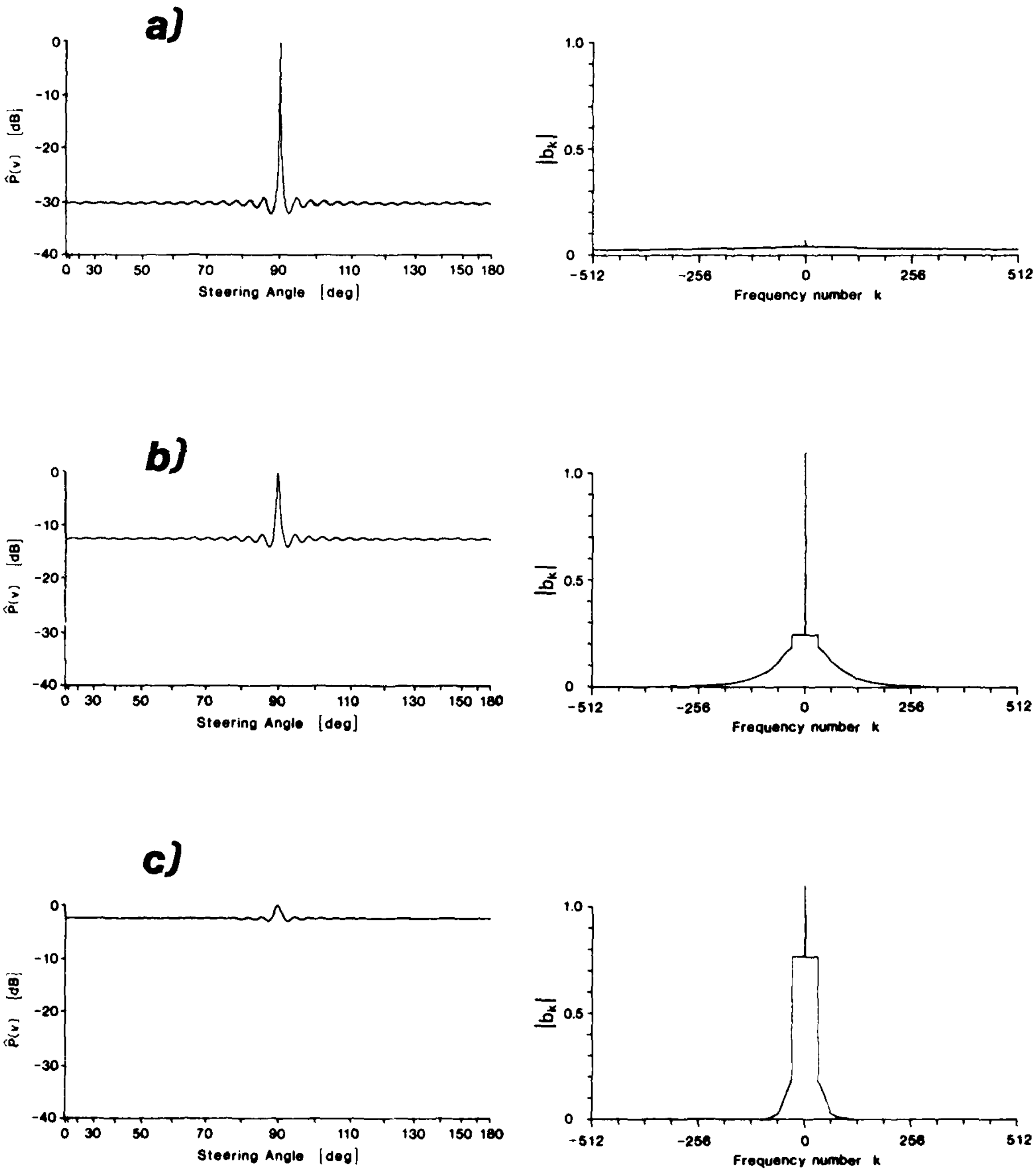

FIG. 5 MAXIMUM ENTROPY BEAMFORMING
a) Signal to noise ratio 0 dB
b) Signal to noise ratio -10 dB
c) Signal to noise ratio -20 dB

Hence this method seems to give good estimates both for very low and for very high σ_s^2/σ_n^2 ratios, and uses the available information consistently.

Therefore

$$\hat{b}_k \approx b_k \;,$$

$$b \approx |k| \leqslant N \;,$$

while it is somehow more imprecise in-between. However it is worthwhile noticing from Eq. 28 that to obtain the peak, the "propagation model" $D(v)$ has to match perfectly the "real propagation", $D(o)$, when $v = 0$. Especially is this the case for high values of σ_s^2/σ_n^2.

3.1.4 Maximum-entropy method

So far, except for the truncation method, none of the methods have explicitly bothered about using the available information in a consistent way, in the sense of explicitly requiring $\hat{b}_k = b_k$ for $|k| \leqq N$. The maximum-entropy method does that. The other fourier-series coefficients are then determined according to the maximum-entropy principle [3]:

$$\max_{\hat{P}(v)} J = \int_{-\frac{1}{2}}^{\frac{1}{2}} dv \ln \hat{P}(v) \quad , \qquad \text{(Eq. 29)}$$

constrained by:

$$r_k = \int_{-\frac{1}{2}}^{\frac{1}{2}} \hat{P}(v)\, e^{-j2\pi kv}\, dv \qquad \text{for} \quad |k| < N \quad . \qquad \text{(Eq. 30)}$$

Also in this case an analytical expression has not been obtained for $\hat{b}_k$ when $|k|>N$. Computer plots (Fig. 5) show for different S/N ratios (a = 0 dB, b = -10 dB, c = -20 dB) the expected behaviour that $\hat{b}_k = b_k = r_k$ for $|k|< N$, and then a smooth decrease in $\hat{b}_k$ approaching zero for large values of k , in agreement with the maximum-entropy principle, (or "minimum a-priori assumption principle").

The $\hat{P}(v)$ plots show excellent "resolution", as the peak is even narrower than the maximum likelihood peak. However, the warning about sensitivity is even more appropriate in this case.

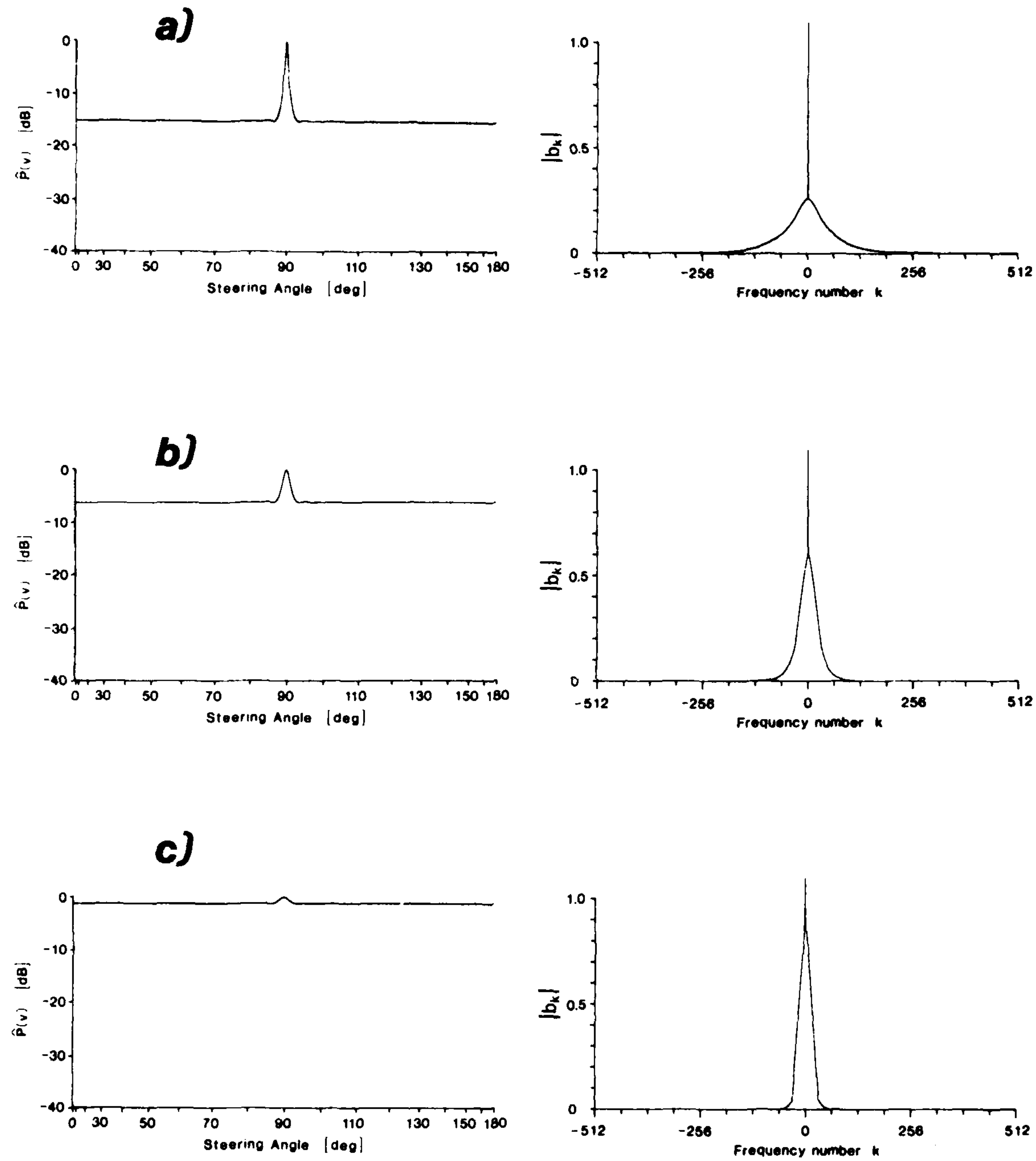

FIG. 4 MAXIMUM LIKELIHOOD BEAMFORMING
a) Signal to noise ratio 0 dB
b) Signal to noise ratio -10 dB
c) Signal to noise ratio -20 dB

The result is:

$$\hat{P}(v) = [D^*(v)\ R^{-1}\ D(v)]^{-1} = \int_{-1/2}^{1/2} du\ P(u) \left| \frac{D^*(v)R^{-1}D(u)}{D^*(v)R^{-1}D(v)} \right|^2 .$$

In our example:

$$\hat{P}(v) = \frac{N\ \sigma_s^2 + \sigma_n^2}{1 + \frac{N\ \sigma_s^2}{\sigma_n^2}\left[1 - \left|\frac{D^*(v)\ D(o)}{N}\right|^2\right]} . \qquad \text{(Eq. 28)}$$

The corresponding $\hat{b}_k$ is not found analytically, but is shown in Fig. 4 together with $\hat{p}(v)$ as a function of $S/N = \sigma_s^2/\sigma_n^2$ for $N = 32$.

The figure shows that the method assigns a value $\hat{b}_k \neq 0$ for $|k| \geqq N$, in such a way that there is a smooth, rounded transition near $|k| = N$, depending on σ_s^2/σ_n^2.

This results both in low or almost no oscillations (no sidelobes), and a narrow peak at the position of the source. Hence it is a big improvement over conventional beamforming, in the sense that it is a pre-whitened matched filter. It takes into account the actual sound-field distribution, to the extent that the matrix R can "image" it. The disadvantage is of course that the available information is still not used in a consistent way, because $\hat{b}_k \neq b_k$ for $k < N$.

To gain some insight, we consider some limiting cases, $\sigma_s^2 \to 0$ and $\sigma_s^2 \to \infty$ for fixed value of σ_n^2 . The first case is trivial, because $b_k = b_k$ for all values of k :

$$\hat{b}_k = \sigma_n^2 , \qquad k = 0$$
$$= 0 . \qquad k \neq 0$$

As $\sigma_s^2 >> \sigma_n^2$, the peak in the P(v) plots becomes both higher and more narrow (at -3 dB below peak level as an example) with increasing σ_s^2 . The $\hat{b}_k$ plots show that the transition from $\hat{b}_k \approx \hat{b}_k$ to $\hat{b}_k \approx 0$ happens both much slower and at increasingly higher k levels.

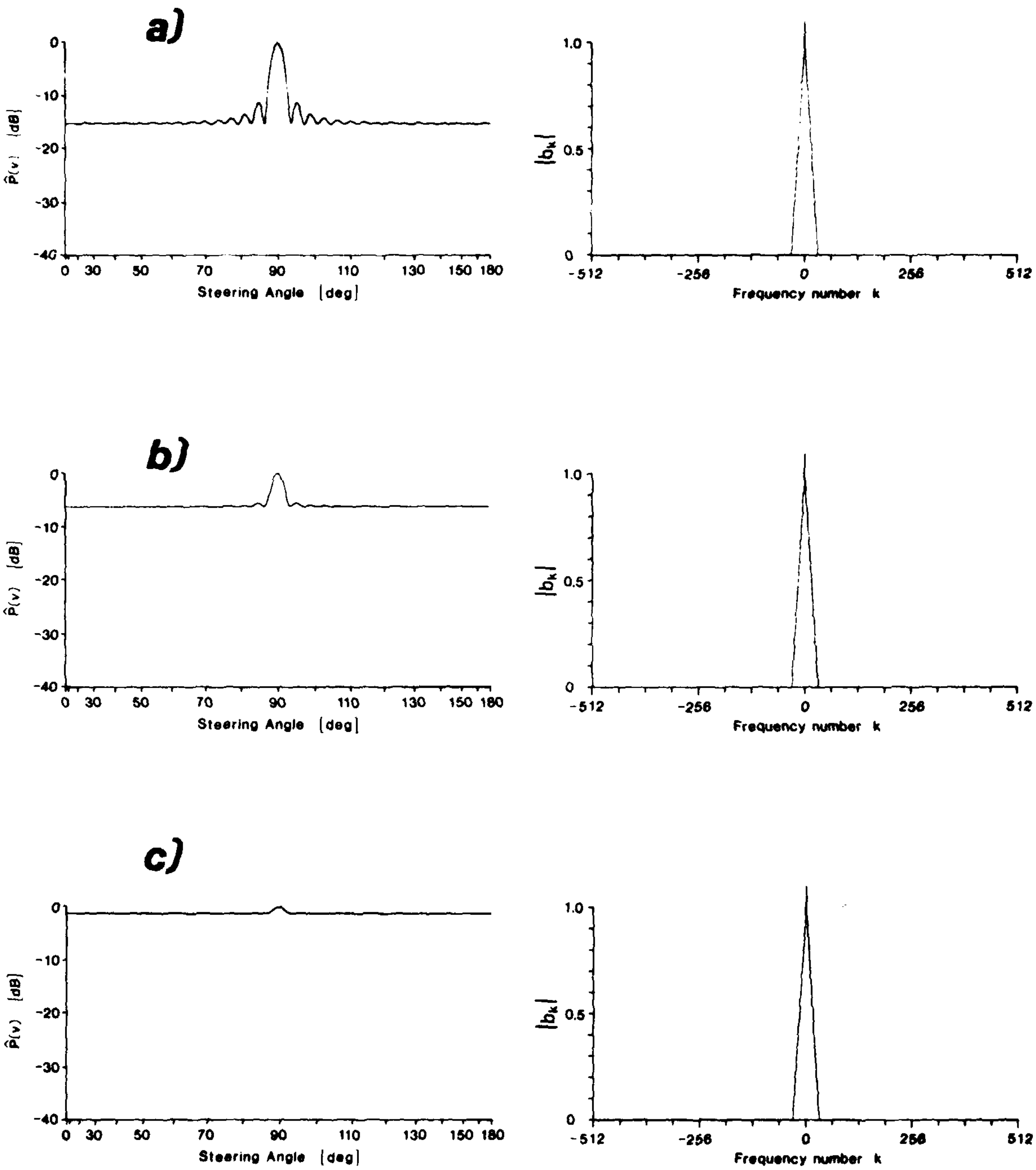

FIG. 3 CONVENTIONAL BEAMFORMING
a) Signal to noise ratio 0 dB
b) Signal to noise ratio -10 dB
c) Signal to noise ratio -20 dB

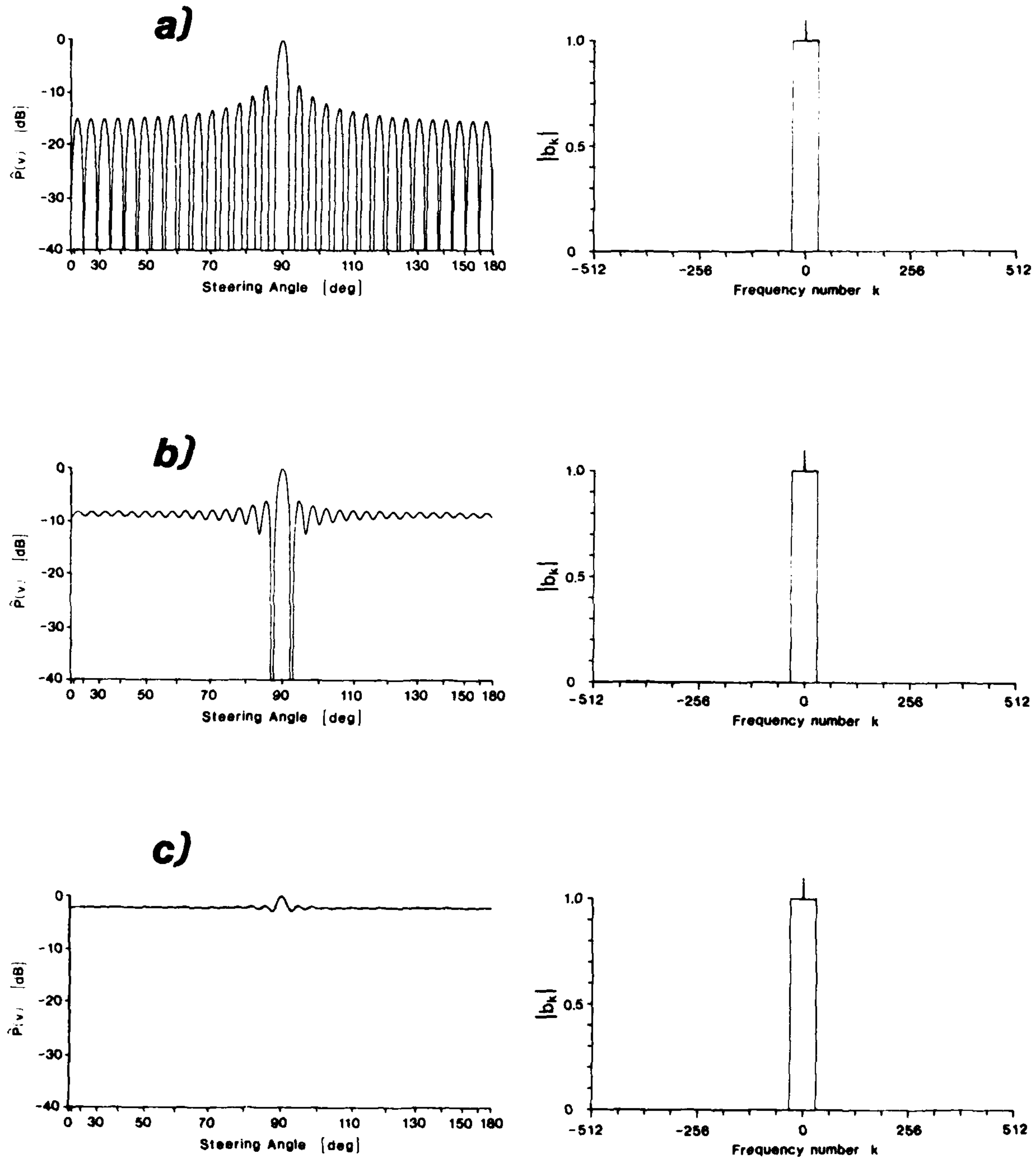

FIG. 2 *TRUNCATED FOURIER SYSTEM*
a) Signal to noise ratio 0 dB
b) Signal to noise ratio -10 dB
c) Signal to noise ratio -20 dB

As seen, this method is using the available information in a consistent way, $\hat{b}_k = r_k$ for $|k| < N$, but the effect of truncation creates heavy oscillations in the estimate of $P(v)$. It might even result in some negative estimates of $P(v)$ for some values of v, depending on the "signal-to-noise" value, σ_s^2 / σ_n^2. Hence the method has certain disadvantages. Figure 2 shows a plot of $|\hat{b}_k|$ and $\hat{P}(v)$ for different signal-to-noise ratios.

3.1.2 The conventional beamformer (the matched filter)

The expected normalized power output of conventional beamforming is given by:

$$\begin{aligned}\hat{P}(v) &= \frac{1}{N} D^*(v)\, R\, D(v) \\ &= \sigma_s^2 \cdot \frac{1}{N} \left| D^*(v) \cdot D(o) \right|^2 + \sigma_n^2 \quad , \\ &= \sigma_s^2 \cdot \frac{1}{N} \left[\frac{\sin(\pi N v)}{\sin(\pi v)} \right]^2 + \sigma_n^2 \; .\end{aligned} \qquad \text{(Eq. 24)}$$

$$\begin{aligned}\hat{b}_k &= (N - |k|) \cdot b_k \quad , \qquad & |k| < N \\ &= 0 & |k| > N\end{aligned} \qquad \text{(Eq. 25)}$$

As seen, this method not only truncates the fourier series but also weights the known values with a triangular window, the so-called Bartlett window. This has the advantage of avoiding high oscillating levels (sidelobe levels), but at the same time broadens the peak of our point source (giving a broad main lobe). Another disadvantage of the weighting is an inconsistent use of available information. As a last comment, this method always yields non-negative estimates of sound-field density. Plots of $|\hat{b}_k|$ and $\hat{P}(v)$ are given in Fig. 3 for different signal-to-noise ratios.

3.1.3 Maximum-likelihood beamforming

The name of this method is misleading, as it is in reality a minimum expected-power beamforming method, constrained by constant gain in the beam-steering direction:

$$\hat{P}(v) = \min_{W(v)} W^*(v)\, R\, W(v) = \min_{W(v)} \int_{-\frac{1}{2}}^{\frac{1}{2}} du\, P(u) \left| W^*(v)\, D(u) \right|^2 , \qquad \text{(Eq. 26)}$$

constrained by:

$$W^*(v)\, D(v) = \text{constant} = N \quad . \qquad \text{(Eq. 27)}$$

From Eqs. 18 and 19, we can say:

- $P_1(v)$ is completely determined by R, and vice versa.
- $P_2(v)$ is completely independent of R, and vice versa.
- As $P(v) = P_1(v) + P_2(v)$, the sound field density can never be uniquely determined from the cross-correlation matrix R alone.

This means that to obtain a unique estimation of $P(v)$, either some additional information is needed or some estimation criteria, constraints, or principles have to be applied. This is, of course, the reason that we have such a jungle of estimation methods and proposals, more or less consistent with the real facts.

In the following sections we will study four methods for estimating $P(v)$, considering the example of a point source in white noise:

$$P(v) = \sigma_s^2\ \delta(v) + \sigma_n^2 \quad , \qquad \text{(Eq. 20)}$$

$$b_k = \int_{-\frac{1}{2}}^{\frac{1}{2}} P(v)\ e^{j2\pi kv}\ dv = \sigma_s^2 + \sigma_n^2 \quad , \quad k = 0 \qquad \text{(Eq. 21)}$$

$$= \sigma_s^2 \quad . \qquad k \neq 0$$

3.1.1 Truncated fourier series method

The truncated method uses the available information about the fourier series of $P(v)$ and neglects the rest:

$$\hat{b}_k = b_k = \sigma_s^2 + \sigma_n^2 \quad , \qquad k = 0$$

$$= \sigma_s^2 \quad , \qquad k \neq 0 \ , \ |k| < N \qquad \text{(Eq. 22)}$$

$$= 0 \quad . \qquad |k| \geq N$$

The corresponding estimate of $P(v)$ is:

$$\hat{P}(v) = P_1(v) = \sum_{k=1-N}^{N-1} \hat{b}_k\ e^{-j2\pi kv}$$

(Eq. 23)

$$= \sigma_s^2 \cdot \frac{\sin[\pi\ (2N-1)\ v]}{\sin(\pi\ v)} + \sigma_n^2$$

Substituting Eq. 11 into Eq. 10 yields:

$$r_k = \int_{-\frac{1}{2}}^{\frac{1}{2}} dv \sum_{\ell=-\infty}^{\infty} b_\ell \, e^{-j2\pi\ell v} \, e^{j2\pi k v} = b_k \quad . \quad |k| < N \tag{Eq. 13}$$

This shows that the elements of diagonal k is equal to the k-th fourier coefficient of the sound-field density.

To see the full consequence of this, we split $P(v)$ into two terms

$$P(v) = P_1(v) + P_2(v) \quad , \tag{Eq. 14}$$

in which

$$P_1(v) = \sum_{\ell=1-N}^{N-1} b_\ell \, e^{-j2\pi\ell v} \quad , \tag{Eq. 15}$$

$$P_2(v) = \sum_{|\ell| \geqq N} b_\ell \, e^{-j2\pi\ell v} \quad . \tag{Eq. 16}$$

From this follows:

$$R_1 = \int_{-\frac{1}{2}}^{\frac{1}{2}} dv \; P_1(v) \; D(v) \; D^*(v) = R \tag{Eq. 17}$$

$$R_2 = \int_{-\frac{1}{2}}^{\frac{1}{2}} dv \; P_2(v) \; D(v) \; D^*(v) = 0 \quad , \tag{Eq. 18}$$

or in words:

- the matrix R is completely determined by $P_1(v)$.
- the matrix R is completely independent of $P_2(v)$.

Also, by combining Eqs. 13 and 15:

$$P_1(v) = \sum_{\ell=1-N}^{N-1} r_\ell \, e^{-j2\pi\ell v} \quad . \tag{Eq. 19}$$

For our linear equi-spaced array it is convenient to change the variable

$$2\pi \cdot v = -k \cdot a \cdot \cos\beta \quad , \tag{Eq. 7}$$

which yields:

$$R = \int_{-a/\lambda}^{a/\lambda} dv\; P(v)\; D(v)\; D^*(v) \quad . \tag{Eq. 8}$$

We also include the virtual field, thereby obtaining:

$$R = \int_{-\frac{1}{2}}^{\frac{1}{2}} dv\; P(v)\; D(v)\; D^*(v) \quad . \tag{Eq. 9}$$

R is always a positive, semi-definite, Hermitian matrix

$$R^* = R \qquad \text{(Hermitian)}$$

$$y^*Ry = \int_{-\frac{1}{2}}^{\frac{1}{2}} dv\; P(v) \left| y^*D(v) \right|^2 \geqq 0 \quad \text{(positive semi-definite)} \quad .$$

For a linear, equidistant array, R defined by Eq. 9 is also Toeplitz:

$$r_{m,n} = \int_{-\frac{1}{2}}^{\frac{1}{2}} dv\; P(v)\; e^{-j2\pi mv}\; e^{+j2\pi nv} = r_{m-n} = r_k \quad , \quad |k| < N \tag{Eq. 10}$$

where $k = 0$ indicates an element on the main diagonal; $k = N-1$ is the element in the lower left corner and $k = 1-N$ is the one in the upper right corner of the R matrix.

We now turn to a quite important problem, judging by the numbers of papers published in the open literature: how well can we estimate the sound-field density $P(v)$ from a knowledge of R ? This is exactly the same problem as estimating the power spectrum $P(f)$ from an estimate of the autocorrelation.

To see this, we write $P(v)$ as a fourier series:

$$P(v) = \sum_{\ell=-\infty}^{\infty} b_\ell\; e^{-j2\pi \ell v} \quad , \tag{Eq. 11}$$

where, since $P(v)$ is a real quantity,

$$b_{-\ell} = b_\ell^* \quad . \tag{Eq. 12}$$

We know that this model is not correct in the real stratified ocean. It seems to be quite a good model for modestly sized arrays, but as soon as the ambition for higher resolution leads to both larger arrays and higher resolution methods, one might be heading for problems.

Improved propagation models have clearly shown that the horizontally layered propagation channel consisting of the surface, ocean, bottom, and sub-bottoms, gives rise to several modes, each having different horizontal wavenumbers. As soon as this difference is sensed by the combination of sufficiently larger array and sufficiently high-resolution method, the mismatch between reality and the plane-wavefront model might cause serious errors in parameter estimation.

In the next section we therefore study what can be done with the normal-mode model in order to improve the situation, and we address eventual problems in using it.

4 NORMAL-MODE PROPAGATION MODEL

4.1 Principles

In this section we substitute the plane-wavefront source vector $D(\beta)$, with the compound normal-mode source vector:

$$F(\beta) = \sum_{m=1}^{M} a_m D(\beta_m) \quad , \qquad \text{(Eq. 50)}$$

where β_m is defined by:

$$k \cos \beta_m = k_m \cos \beta \quad . \qquad \text{(Eq. 51)}$$

One might think that it is simple to use $F(\beta)$ instead of $D(\beta)$ in all the traditional methods. But there is one important difference. While $D(\beta)$ depends on only one environmental parameter, which is the local sound speed around the array, $F(\beta)$ has a much more complicated dependence on the channel parameters. In fact, in order to know the mode weight a_m, or more generally, the mode vector:

$$A = [\ a_1, \ldots, a_M\ ]^T \quad ,$$

one has to know not only the local normal-mode characterization of the channel, but the channel parameters all the way from the source to the receiver. It is a highly unrealistic assumption that the mapping of the ocean and the bottom parameters will ever be so detailed as to allow an a priori calculation of A with the necessary accuracy. Imagine only the problem of obtaining a satisfactory value for the relative phase of the A elements. To do so, we really need to know the range to the source quite well. As to the value of the relative amplitude, this depends not only on the relative attenuation of the modes, but also on the depth of the source. And this even neglects the effect of any randomness in the channel, or any movement of the source or receiver.

The above arguments have convinced the authors that the model vector A can never be calculated satisfactorily from scratch (channel parameters); hence it is not possible to obtain an a priori source vector model $F(\beta)$. So far the only information we have is a structure for $F(\beta)$, given by Eq. 50.

Looking to the modal source vectors $D(\beta_m)$, they depend on the horizontal wavenumbers k_m. But as the source vectors really express relative phase rotation (or phase delay), it is the local horizontal wavenumbers that are required. Hence it does not really matter if the channel is range-dependent between source and receiver, as long as we know the local channel

parameters quite well. We will make the following assumption:

- The local environmental parameters are known so well that the horizontal wavenumbers can be calculated with sufficient accuracy.

What we mean with "sufficient accuracy" depends on the array size and processing method.

At this point we have obtained a structuring of $F(\beta)$, according to Eq. 50, in which all the source vectors, $D(\beta_m)$, except the bearing, are known but where the mode weights, a_m, are unknown and have to be estimated. On one hand this gives many more parameters to estimate than for the plane-wavefront model. On the other hand, it results in a non-Toeplitz matrix containing much more information.

It seems obvious that methods requiring a priori knowledge of the source vector are useless in this case. Hence, we concentrate on the orthogonal decomposition method in the next section.

4.2 Orthogonal Decomposition Method

It still seems convenient to split the cross-correlation matrix into two terms, one caused by discrete point sources and one by continuous (distributed) ones, such as the sea surface. The discrete term is:

$$R_d = \sum_{i=0}^{T-1} \sigma_i^2 \, F_i(\beta_i) \, F_i^*(\beta_i) \quad , \qquad \text{(Eq. 52)}$$

$$= \sum_{i=0}^{T-1} \sigma_i^2 \, G(\beta_i)\cdot A_i \cdot A_i^* \cdot G^*(\beta_i) \quad ,$$

where we have introduced:

$$F_i(\beta_i) = \sum_{n=o}^{M-1} a_{m,i} \, D(\beta_{m,i}) = G(\beta_i)\cdot A_i \quad , \qquad \text{(Eq. 53)}$$

$$G_i = [D_{o,i} \dots D_{m,i} \dots D_{M-1},i] \qquad \text{(Eq. 54)}$$

$$A_i = [a_{o,i} \dots a_{m,i} \dots a_{M-1},i]^T \quad , \qquad \text{(Eq. 55)}$$

in which A_i, σ_i^2 and β_i are the unknown quantities to be determined. Notice that the rank of the matrix is T, every source contributing with rank 1.

A similar model of the continuous part would be forbiddingly complex. Luckily, in this case the intermodal terms have a tendency to cancel each other [10]. Therefore the structure again simplifies to one that is identical to the plane-wavefront case:

$$R_c = \int dv \; P_c(v) \; D(v) \; D^*(v) \quad ,$$

$$= \sum_{k=1-K}^{K-1} b_{k,c} \cdot I_k \quad . \qquad \text{(Eq. 56)}$$

The total model is then:

$$R = \sum_{i=o}^{T-1} \sigma_i^2 \; G(\beta_i) \; A_i \; A_i^* \; G^*(\beta_i) + \sum_{k=1-K}^{K-1} b_{k,c} \; I_k \quad . \qquad \text{(Eq. 57)}$$

Hence we can again use the same estimation criterion, Eqs. 47 and 48, and the same algorithm as in the plane-wavefront case of Sect. 3.1.5. The rank of R_d does not change with the change of model as long as the modal source vectors add coherently.

The problem is now to estimate the unknown parameters of R_d by matching $\hat{R}_d$ and R_d, both assumed to have reduced rank T<N:

One procedure to determine those parameters is given in App. B.

5 NORMAL-MODE PROPAGATION MODEL WITH DYNAMIC SOURCE/RECEIVER GEOMETRY

As long as the source and the receiver do not move, we have assumed that there are no problems in obtaining a good estimate of the cross-correlation matrix, using time averaging. The introduction of dynamic source/receiver geometry, however, introduces an essential difference between the classical plane-wave source vector (delay vector) and the normal-mode source vector.

In the plane-wave case, the source vector normally changes quite slowly when the source is in the far field. Hence it might still be possible to obtain quite a good estimate of the matrix, given that the receiver system does not change course.

It is quite a different situation with the normal-mode source vector. As mentioned earlier, this consists of a weighted sum of source vectors for the individual modes. These individual source vectors behave exactly like the plane-wave source vector. The relative phases of the weights, however, rotate with time, causing the individual vectors to combine into a resulting source vector that changes quite drastically with time. The deterministic relative phase rotations in the weights have two effects:

- The modes have different horizontal wave vectors, such that a displacement in space results in relative phase rotation.

- The modes have different doppler shifts, such that displacement in time results in relative phase rotation.

The end result is that the stationary condition for replacing ensemble-averaging with time-averaging is not well satisfied. There seem to be several plausible ways to resolve this dilemma. Three of them are outlined in the next sections.

5.1 Update of Matrix with Stationary Model

We have seen that the normal-mode source vector changes with time in a dynamic source/receiver situation. Let us see what happens to the contribution to the vector from such a source if we still prefer straightforward time-averaging.

To repeat, the data vector X can be written:

$$X = s \sum_m a_m D_m$$

$$\hat{R} = \overline{ss^* \sum_m \sum_n a_m a_n^* D_m D_n^*} \quad ;$$

s is independent of the weights a and D is assumed not to change over the averaging time:

$$\hat{R} = \sigma_s^2 \sum_m \sum_n \overline{a_m a_n^*} D_m D_n^* \quad .$$

Because of the relative phase between the weights, $\overline{a_m a_n^*} \approx 0$ for $m \neq n$, and

$$\hat{R} \approx \sigma_s^2 \sum_{m=1}^{M} \overline{a_m a_m^*} \cdot D_m D_m^* \quad .$$

This expression corresponds to M independent point sources using a plane-wave model.

If the array is large enough or has enough vertical extension to resolve the mode, this matrix model might be useful. What is lost is information about the weight phase, which is sometimes of interest. Nevertheless the approach is simple and might be given further thought. With several sources present, however, it might be difficult to identify the contributions from the different sources.

5.2 Update of Matrix with Non-stationary Model

A common way to handle a non-stationary situation is to use a state/space approach, which in the linear, gaussian case results in the well-known Kalman-Bucy filter. Even in many non-linear and non-gaussian cases it has been used with some success. What are required in this approach are a state model and a measurement model. Very little work has been reported on the use of such a method to estimate (track) the cross-correlation matrix itself [11], but a straightforward Kalman-Bucy filter approach does not seem feasible. Some work is being done [12] on the use of an input/output formulation (the opposite of the state/space formulation), the so-called scattering framework for estimation, to handle non-stationarities but results have not yet been published.

5.3 Abandonment of Matrix

When dealing with zero-mean, gaussian, stationary, stochastic processes in linear systems, the cross-correlation matrix yields all the available information.

Sections 5.1 and 5.2 indicate that maintaining an approach based on the cross-correlation matrix might lead to difficulties in dynamic situations. This is even more so if non-linearities and non-gaussian processes enter the picture. Therefore an increasing amount of attention is being given to more general approaches capable of handling quite complex situations and many books and conferences are partly or completely occupied with these

problems [13-16]. As a result of its success in the linear gaussian case, the Kalman-Bucy filter, most of the work is based on the state/space approach.

Work in this direction initially led to large problems dealing with non-linear, stochastic, partial differential equations (PDE). This situation has improved drastically in recent years, going through an intermediate phase of linear stochastic PDE to the latest progress in which a linear deterministic PDE is parametrized by the stochastic process represented by the measurements [17].

SACLANT ASW Research Centre has also done some work in this general direction for the special case of target- motion analysis [18-20].

It might be that to gain maximum benefit from improved propagation models, of which the normal mode is an important example, one has to turn to such general methods.

CONCLUSION

It is quite common in passive sonar data processing to:

- Apply a plane wavefront propagation model.
- Assume stationary processes.
- Let the cross-correlation matrix of the array data play an important part in the processing.

For a horizontal array of modest size the three points above are quite reasonable. Shallow-water sea trials with towed arrays have demonstrated that the wavefront model seems almost perfect for such arrays. The combination of modest size and a plane-wavefront model makes the processing quite robust with respect to the stationary assumption. Hence the cross-correlation matrix can be estimated quite well without major problems.

The main problem with a small array is its limited resolution. The sound-field density can be estimated only with fundamental uncertainties, in spite of high-resolution methods. The necessary information is not in the data.

Increasing the array size leads to a chain reaction of consequences. The first consequence might be that a plane-wavefront model is inconsistent with the data. A more refined propagation model is then necessary. At least for low-frequency, shallow-water conditions, a normal-mode propagation model seems a natural choice, and was attempted for use in this report.

This leads directly to the second consequence, that better knowledge of the environmental parameters is necessary. The plane-wavefront model required only the knowledge of the sound speed around the array. As a minimum (light modelling) the normal-mode model requires the environmental knowledge needed to calculate the modal horizontal wavenumbers [7]. In fact we may never know the channel well enough to apply some traditional processing schemes. For instance, with conventional beamforming in which the steering vector of the plane-wavefront model is replaced by a vector that has been combined over all the modes, we might know the individual modal vectors (which depend only on the horizontal wavenumbers around the array) but not the combining weights.

Hence the third consequence is that traditional processing methods might be ruled out, including also maximum-likelihood and maximum-entropy beamforming. One is forced to such methods as orthogonal decomposition.

The fourth consequence is that the combined effect of a larger array and refined modelling makes the processing more sensitive to non-stationarity. As an example, the source vector of the normal-mode model (replacing the steering vector of the plane-wavefront model) is sensitive to changes in geometry caused by movement of source and/or receiver. This means that a non-stationary cross-correlation matrix R is required, making it difficult, perhaps impossible, to obtain a good estimate of this central quantity in present data-processing methods.

Hence the fifth consequence is that processing based on the matrix R might be abandoned and replaced by methods that take into account the modelling of the dynamic nature of the involved processes. One obvious candidate is a state/space approach, of which the Kalman-Bucy filter is the best-known member.

We might be at a crucial point in research on (passive) sonar data processing, a point where data-processing researchers, sound-propagation model researchers, and research managers need to work coherently together for the common aim of improving passive sonar system performance.

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APPENDICES

APPENDIX A

PARAMETER ESTIMATION TECHNIQUE FOR THE ORTHOGONAL DECOMPOSITION METHOD

Let R_ℓ , $\ell = 0, 1, \ldots L$, be constant Hermitian matrices:

$$R = R_0 - \sum_{\ell=1}^{L} b_\ell R_\ell = \sum_{i=1}^{N} \lambda_i E_i E_i^* \quad , \qquad L < N \qquad \text{(Eq. A.1)}$$

where λ_i and E_i , $i=1, \ldots N$, are eigenvalues and eigenvectors of the Hermitian matrix R . Therefore all the eigenvalues are real and, by convention, ordered by decreasing value.

The problem is to minimize J :

$$J = \sum_{i=N-P+1}^{N} \lambda^2{}_i \qquad\qquad P < N \qquad \text{(Eq. A.2)}$$

with respect to the parameter vector B:

$$B = [b_1 , b_2 \ldots b_L]^T \quad . \qquad \text{(Eq. A.3)}$$

One iterative algorithm for this non-linear problem is:

$$B(n+1) = B(n) - \alpha(\nabla_B^2 J + \mu I)^{-1} \nabla_B J \quad , \qquad \text{(Eq. A.4)}$$

where the gradient of J with respect to B is:

$$G = \nabla_B J = \left[\frac{\partial J}{\partial b_1}, \ldots \frac{\partial J}{\partial b_L} \right]^T \qquad \text{(Eq. A.5)}$$

and the so called Hess matrix of J with respect to B is:

$$H = \Delta_B^2 J = \begin{bmatrix} \frac{\partial^2 J}{\partial b_1 \partial b_1} & \cdots & \frac{\partial^2 J}{\partial b_1 \partial b_L} \\ \vdots & & \vdots \\ \frac{\partial^2 J}{\partial b_L \partial b_1} & \cdots & \frac{\partial^2 J}{\partial b_L \partial b_L} \end{bmatrix} \qquad \text{(Eq. A.6)}$$

Our purpose in this appendix is to find expressions for the elements of G and H. We make the following definitions:

$$D_m(\ ) = \frac{\partial(\ )}{\partial b_m} , \qquad \text{(Eq. A.7)}$$

$$\beta_{imj} = E_i^* D_m(E_j) , \qquad \text{(Eq. A.8)}$$

$$\gamma_{imj} = E_i^* R_m E_j . \qquad \text{(Eq. A.9)}$$

Using the operator Eq. A.7 on Eq. A.2 yields:

$$D_m(J) = 2 \sum_{i=N-P+1}^{N} \lambda_i D_m(\lambda_i) , \qquad \text{(Eq. A.10)}$$

$$D_k(D_m(J)) = 2 \sum_{i=N-P+1}^{N} D_k(\lambda_i) D_m(\lambda_i) + \lambda_i D_k(D_m(\lambda_i)) . \qquad \text{(Eq. A 11)}$$

We now use many of the properties of a Hermitian matrix and its eigenvalues and eigenvectors:

$$(R - \lambda_i I) E_i = 0 \qquad \text{(Eq. A.12)}$$

Using Eq. A.1 in Eq. A.12, and operating with D_m , gives:

$$(- R_m - D_m(\lambda_i) I) E_i + (R - \lambda_i I) D_m(E_i) = 0 . \qquad \text{(Eq. A.13)}$$

Premultiplying with E^*_j gives:

$$-E_j^* R_m E_i - D_m(\lambda_i) E_j^* E_i + (E_j^* R - \lambda_i E_j^*) D_m(E_i) = 0 . \qquad \text{(Eq. A.14)}$$

A Hermitian matrix has orthonormal eigenvectors and real eigenvalues:

$$- E_j^* R_m E_i - D_m(\lambda_i) \partial_{ij} + (\lambda_j - \lambda_i) E_j^* D_m(E_i) = 0 . \qquad \text{(Eq. A.15)}$$

For $i = j$:

$$D_m(\lambda_i) = - E_i^* R_m E_i = - \gamma_{imi} , \qquad \text{(Eq. A.16)}$$

$$D_k(D_m(\lambda_i)) = - D_k(E_i^*) R_m E_i - E_i^* R_m D_k(E_i) . \qquad \text{(Eq. A.17)}$$

Equation A.13 can now be written with a more compact notation:

$$(\lambda_j - \lambda_i)\, \beta_{jmi} = \gamma_{imi}\, \gamma_{ij} \quad . \tag{Eq. A.18}$$

From the orthonormal property of eigenvectors:

$$E_i^* E_j = \delta_{ij} \quad , \tag{Eq. A.19}$$

$$D_m(E_i^*)\, E_j + E_i^* D_m(E_j) = 0 \quad , \tag{Eq. A.20}$$

$$E_i^* D_m(E_j) = - D_m(E_i^*)\, E_j = \beta_{imj} \quad . \tag{Eq. A.21}$$

$D_m(E_j)$ is in a vector space spanned by the N eigenvectors. It can easily be shown that:

$$D_m(E_j) = \sum_{i=1}^{N} \beta_{imj} E_i \quad . \tag{Eq. A.22}$$

by premultiplying with E^*_n and using Eq. A.19.

Equation A.17 can now be written:

$$D_k(D_m(\lambda_i)) = \sum_{j=1}^{N} \beta_{ikj}\gamma_{jmi} - \gamma_{imj}\beta_{jki} \quad . \tag{Eq. A.23}$$

Furthermore, Eqs. A.10 and A.11 are:

$$D_m(J) = -2 \sum_{i=N-P+1}^{N} \gamma_{imi} \quad , \tag{Eq. A.24}$$

$$D_k(D_m(J)) = 2 \sum_{i=N-P+1}^{N} \gamma_{iki}\gamma_{imi} \tag{Eq. A.25}$$

$$+ 2 \sum_{i=N-P+1}^{N} \sum_{j=1}^{N} \lambda_i \beta_{ikj}\gamma_{jmi} - \lambda_i \beta_{jki}\gamma_{imj} \quad .$$

The last equation (Eq. A.25) has to be manipulated quite a lot:

$$D_k(D_m(J)) = 2 \sum_{i=N-P+1}^{N} \gamma_{iki}\gamma_{imi}$$

$$+ 2 \sum_{i=N-P+1}^{N} \sum_{j=1}^{N-P} \lambda_i\beta_{ikj}\gamma_{jmi} - \lambda_i\beta_{jki}\gamma_{imj} \qquad \text{(Eq. A.26)}$$

$$+ 2 \sum_{i=N-P+1}^{N} \sum_{j=N-P+1}^{N} \lambda_i\beta_{ikj}\gamma_{jmi} - \lambda_j\beta_{ikj}\gamma_{jmi} \quad ,$$

where the indices i and j have been exchanged in the last term of the last line. This can be done because of symmetry. Using Eq. A.18 in Eq. A.26 yields

$$D_k(D_m(J)) = 2 \sum_{i=N-P+1}^{N} \gamma_{iki}\gamma_{imi}$$

$$+ 2 \sum_{i=N-P+1}^{N} \sum_{j=1}^{N-P} \frac{\lambda_i}{\lambda_j - \lambda_i}(\gamma_{jki}\gamma_{imj}+\gamma_{ikj}\gamma_{jmi}) \qquad \text{(Eq. A.27)}$$

$$+ 2 \sum_{i=N-P+1}^{N} \sum_{j=N-P+1}^{N} (\gamma_{ikj} - \gamma_{iki}\delta_{ij}) \; \gamma_{jmi} \quad ,$$

$$D_k(D_m(J)) = \sum_{i=N-P+1}^{N} \sum_{j=1}^{N-P} \frac{2\lambda_i}{\lambda_j - \lambda_i} \; R_e\{\gamma_{ikj}\gamma_{jmi}\}$$

$$+ \sum_{i=N-P+1}^{N} \sum_{j=N-P+1}^{N} 2\,\gamma_{ikj}\gamma_{jmi} \quad , \qquad \text{(Eq. A.28)}$$

$$D_k(D_m(J)) = \sum_{i=N-P+1}^{N} \sum_{j=1}^{N} v_{ij} \; \mathrm{Re}\{\gamma_{ikj}\gamma_{jmi}\} \quad ,$$

where

$$v_{ij} = \frac{4\lambda_i}{\lambda_j - \lambda_i}, \qquad j < N-P \qquad \text{(Eq. A.29)}$$

$$= 2 \quad , \qquad j > N-P \quad .$$

One might have to increase the eigenvalues λ_j, $j=1, \ldots N-P$, slightly, in order to be able to divide by $\lambda_j - \lambda_i$, $j < N-P$, $i > N-P$.

APPENDIX B

GENERALIZED GONIOMETER TECHNIQUE

Let R be a Hermitian matrix:

$$R = F\,F^* = \sum_{i=1}^{T} F_i\,F_i^* = \sum_{i=1}^{T} \sigma_i^2 \; D_i A_i A_i^* D_i^* \;, \qquad \text{(Eq. B.1)}$$

where F_i , $i=1, \ldots T$ are source vectors:

$$F_i = \sigma_i \sum_{m=1}^{M} a_{m,i} D_{m,i} = \sigma_i D_i A_i \;, \qquad \text{(Eq. B.2)}$$

$$F = [F_1 \ldots F_T] \;. \qquad \text{(Eq. B.3)}$$

$D_{m,i}$ is the mode vector for mode m and source i :

$$D_{m,i} = \begin{bmatrix} e^{-jk_m x_1 \cos(\beta_i)} \\ \vdots \\ e^{-jk_m x_N \cos(\beta_i)} \end{bmatrix} , \qquad \text{(Eq. B.4)}$$

$$D_i = [D_{1,i} \ldots D_{m,i}] \;, \qquad \text{(Eq. B.5)}$$

and A_i is the normalized mode amplitude vector:

$$A_i = [a_{1,i} \ldots a_{M,i}]^T \;, \qquad \text{(Eq. B.6)}$$

$$A_i^* A_i = 1 \;. \qquad \text{(Eq. B.7)}$$

We develop some properties for the case:

$$T < N \qquad \text{(Eq. B.8)}$$

$$M \leqslant N-T \;. \qquad \text{(Eq. B.9)}$$

The matrix R has N orthonormal eigenvectors E_n , and N real non-negative eigenvalues λ_n , $n=1 \ldots N$. The eigenvalues are indexed in decreasing order. From Eq. B.7 it follows that:

$$\lambda_n = 0 \qquad n=T+1, \ldots N \;, \qquad \text{(Eq. B.10)}$$

such that:

$$R = E_D \Lambda_D E_D^* \quad , \qquad \text{(Eq. B.11)}$$

where:

$$E_D = [E_1 \ldots E_T] \quad , \qquad \text{(Eq. B.12)}$$

$$\Lambda_D = \begin{bmatrix} \lambda_1 & 0 & \ldots & 0 \\ 0 & \lambda_2 & \ldots & 0 \\ & & \ddots & \\ 0 & 0 & \ldots & \lambda_T \end{bmatrix} \quad , \qquad \text{(Eq. B.13)}$$

E_D spans a T-dimensional subspace S_D . The source vectors F_i , i=1,...T, will be linear combinations of these eigenvectors:

$$F_i = \sum_{n=1}^{T} z_{n,i} \lambda_n^{1/2} E_n = E_D \Lambda_D^{1/2} Z_i \quad , \quad i=1, \ldots T \quad , \qquad \text{(Eq. B.14)}$$

where:

$$Z_i = [z_{1,i} \ldots z_{T,i}] \qquad \text{(Eq. B.15)}$$

We can write Eq. B.14 more compactly as

$$F = E_D \Lambda_D^{1/2} Z \quad , \qquad \text{(Eq. B.16)}$$

where:

$$Z = [Z_1 \ldots Z_T] \quad . \qquad \text{(Eq. B.17)}$$

Inserting Eq. B.16 into Eq. B.1 gives:

$$R = E_D \Lambda_D^{1/2} Z Z^* \Lambda_D^{1/2} E_D^* \quad . \qquad \text{(Eq. B.18)}$$

This will coincide with Eq. B.11 only if:

$$Z Z^* = I \quad . \qquad \text{(Eq. B.19)}$$

As the T source vectors defining F are independent (discrete sources), Z is non-singular, and has an inverse:

$$Z^{-1} = Z^* \quad , \tag{Eq. B.20}$$

$$Z^* Z = I \quad . \tag{Eq. B.21}$$

Hence Z has to be an unitary matrix, see also <6>.

The remaining eigenvectors will define an (N-T)-dimensional subspace S that is orthogonal to the source vectors:

$$E_\perp^* F_i = 0 \quad , \qquad i=1, \ldots T \tag{Eq. B.22}$$

where:

$$E_\perp = [E_{T+1} \ldots E_N] \quad . \tag{Eq. B.23}$$

Substituting Eq. B.2 into Eq. B.22 yields:

$$E_\perp^* \; D_i \; A_i = 0 \quad , \qquad i=1, \ldots T \quad . \tag{Eq. B.24}$$

Let $\hat{R}$ be a N-dimensional hermitian matrix of rank T. We want to match it as well as possible to a matrix R given by the model of Eq. B.1. The parameters to be estimated are σ_i, β_i, and A_i, $i=1, \ldots T$.

For K values of β (assuming β is properly quantized), we try to satisfy Eq. B.24, using eigenvectors from R. Hence we want to minimize:

$$J_k = |\bar{E}_\perp^* \, D_k \, A_k|^2 \qquad k=1, \ldots K \tag{Eq. B.25}$$

with respect to A_k, while at the same time obeying Eq. B.7. The solution is the eigenvector of $D_k^* \; \bar{E}_\perp \bar{E}_\perp^* \; D_k$ with the smallest eigenvalue ε_k:

$$D_k^* \; \bar{E}_\perp \bar{E}_\perp^* \, D_k \, \bar{A}_k = \varepsilon_k \bar{A}_k \quad , \qquad k=1, \ldots K \tag{Eq. B.26}$$

$$J_k = \varepsilon_k \quad , \qquad k=1, \ldots K \quad . \tag{Eq. B.27}$$

Let us plot J_k as a function of β_k, and assume (and hope) that this plot has T minimal points (and hope that they are close to zero). These points define our estimates β_i and A_i, $i=1, \ldots T$. This method may be called a generalized goniometer technique, see <7>. One way to obtain an

estimate σ_i , i=1, ... T, is to combine Eqs. B.2 and B.14:

$$\bar{\sigma}_i D_i \bar{A}_i = \bar{E}_D \bar{\Lambda}_D^{\frac{1}{2}} \bar{Z}_i \quad . \tag{Eq. B.28}$$

Premultiplying Eq. B.28 with $\bar{\Lambda}_D^{-\frac{1}{2}} \bar{E}_D^*$ gives

$$\bar{Z}_i = \bar{\Lambda}_D^{-\frac{1}{2}} \bar{E}_D^* D_i \bar{A}_i \bar{\sigma}_i \quad . \tag{Eq. B.29}$$

From Eq. B.21 it follows that $\bar{Z}_i$ should have unit length:

$$1 = \bar{Z}_i^* \bar{Z}_i = A_i^* D_i^* \bar{E}_D \bar{\Lambda}_D^{-1} \bar{E}_D^* D_i \bar{A}_i \bar{\sigma}_i^2 \quad , \tag{Eq. B.30}$$

$$\bar{\sigma}_i^2 = \left[\bar{A}_i^* D_D^* \bar{E}_D \bar{\Lambda}_D^{-1} \bar{E}_i^* D_i \bar{A}_i\right]^{-1} \quad , \quad i=1, \dots T \; . \tag{Eq. B.31}$$

Another approach is the following:

$$R = \sum_{i=1}^{T} \sigma_i^2 D_i A_i A_i^* D_i^* = \sum_{i=1}^{T} \sigma_i^2 R_i \; . \tag{Eq. B.32}$$

Then, $\bar{\sigma}_i$, i=1, ... T, can be estimated by minimizing the norm:

$$J = \| \bar{R} - R \| = \| \bar{R} - \sum_{i=1}^{T} \sigma_i^2 R_i \| \tag{Eq. B.33}$$

with respect to σ_i , i=1, ... T.

KEYWORDS

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